ABSTRACT

High organic strength waste such as wastewater generated from various industrial activities and sludge generated from industrial and municipal wastewater treatment plant require treatment prior to disposal in order to reduce environment pollution load and also to comply with disposal norms. Waste containing organic impurities in nature must be treated biologically using life forms such as microorganisms, plants etc. Anaerobic digestion is the most suitable treatment method for treating high organic strength waste as compare to other waste treatment methods, because energy in the form of methane is recovered and also treated sludge has good soil conditioning values. Complete mix anaerobic digester is more suitable for treating high organic strength waste as compare to other because of proper heat transfer, proper mixing for maintaining uniformity throughout the reactor. Complete mix reactors are resistant to shock loading and also withstand high organic loading rates. Anaerobic digester suffers stability problem due to accumulation of volatile fatty acids and drop in pH. Once the failure of digester takes place it requires long time and efforts to restart it again. Dynamic modelling and simulation are useful tools for predicting the process stability by studying the behaviour under transient conditions and are also helpful in understanding the process operations.

In the present work attempt has been made to develop simplified dynamic mathematical model for the anaerobic digestion of the sludge. The inhibitory function developed by Andrews (1969) given below was an important investigation considering the inhibitory effect of the unionized volatile fatty acid on methanogen bacteria. The unionized volatile fatty acid act as growth limiting substrate at lower concentration and cause inhibition at higher concentration for microorganism utilizing them as substrate.

The inhibition model was represented

\[ \mu = \frac{\hat{\mu}}{1 + \frac{K_s}{HS} + \frac{HS}{K_i}} \]

where

- \( \mu \) Specific growth rate (day\(^{-1}\))
- \( \hat{\mu} \) Maximum specific growth rate in the absence of inhibition (day\(^{-1}\))
- \( HS \) Unionized substrate concentration (mg/L)
- \( K_s \) Saturation constant (mg/L)
- \( K_i \) Inhibition constant (mg/L)

The model was restricted to fixed pH and only methanogenesis step was considered. Andrews et al. (1971) removed the limitation of fixed pH by considering the interaction between liquid, gas and biological phases and pH variation took place between 6-8. The buffering system was carbon dioxide – bicarbonate. The model was also restricted to methanogenesis step. Hill et al. (1977) developed model for animal waste digestion based on two microbial culture acidogens and methanogens. They also considered the inhibition caused by unionized volatile acid and unionized ammonia on growth kinetics of the methanogens.
The inhibition model was given by

\[
\mu = \frac{\hat{u}}{1 + \frac{K_s}{V_A} + \frac{V_A}{K_i_a} + \frac{NH_3}{K_i_2}}
\]

where

\[\mu\] Specific growth rate (day\(^{-1}\))
\[\hat{u}\] Maximum specific growth rate in the absence of inhibition (day\(^{-1}\))
\[V_A\] Unionized substrate concentration (mg/L)
\[K_s\] Saturation constant (mg/L)
\[K_i_a\] Inhibition coefficient for acids (mg/L)
\[K_i_2\] Inhibition coefficient for ammonia (mg/L)
\[NH_3\] Concentration of unionized ammonia (mg/L)

Havlik et al. (1986) developed model for anaerobic digestion of complex organic substrates and also stated that path of methane generation was either from acetate or via carbon dioxide reduction by hydrogen. The model developed by Moletta et al. (1986) was based on easily fermentable organics such as pea bleaching wastewater and synthetic substrate containing sucrose and organic acid. In the developed model biomass and metabolite production rates were described by distinct relations. No pH variation and no buffering system were assumed in the model. The dynamic model developed by Bello-Mendoza et al. (1998) for anaerobic digestion of sewage sludge described digester behaviour under non-ideal mixing conditions. He considered not any buffering system in the developed model and the growth rate of microorganisms was assumed to be dependent on Monod Kinetics. Hydrolysis and death of microorganisms is described by first order reactions.

In 1977, an International Anaerobic Modelling Task Group was established in Japan as a common platform for the establishment of anaerobic digestion model for complex organic wastes.

The kinetic model developed in the present work describe the anaerobic digestion of sludges considering the hydrolysis of particulate material to soluble compounds by the extracellular enzymes produced by the acidformers; and methanogenesis of volatile acids to methane and carbon dioxide by methogenic bacteria. In most of the cases the first step in anaerobic digestion of wastes depends on the nature of organic impurities. Sometimes fermentation may be the first step if most of the impurities are soluble in nature. The growth and decay of acidogens and methanogens are assumed to depend on Andrews inhibitory function. Hydrolysis of particulate organic compounds is described by first order reaction. The microbial kinetic model expressions are linked to the complete mixed anaerobic sludge digester. Computer simulation of developed mathematical model helps two predict the dynamic behaviour of the developed model under batch mode, transient conditions and also steady state conditions and also helpful in improving design.

The developed modified dynamic mathematical model describes the process operations in more quantitative way which is helpful in designing a better control system to enhance stability, improving performance of anaerobic digester and also optimizing process performance.

**Keywords:** Anaerobic digester, organic waste, sewage sludge, modelling and computer simulation

**INTRODUCTION**

Anaerobic digestion is widely used method for the stabilization of high organic strength waste such as industrial wastewater, municipal wastewater, sewage sludge, and industrial sludge; and also applicable for the stabilization of solid waste. This method is advantageous as compared to other method of stabilization of organic waste such as aerobic because of the production of methane by the decomposition of organic compounds. Due to the complex process operations inside the digester during degradation of complex organic compounds, problem generally occurs due to failure of digester. With the help of dynamic mathematical model, by predicting behaviour under varying conditions performance of digester can be evaluated and helpful in improving design.
The inhibitory function developed by Andrews (1969) has been used by many researchers in predicting behaviour of digester by considering the inhibitory effect on methanogens by unionized volatile fatty acid. Hill et al. (1977) developed mathematical model for anaerobic digestion of animal waste and also considered the inhibitory effect of unionized volatile fatty acids on acidogens and methanogens. Bello-Mendoza et al. (1998) has developed mathematical model of anaerobic digester and simulate its performance under non ideal mixing conditions. Utilization of substrate and growth of acidogens and methanogens is based on Monod kinetic model.

**MATHEMATICAL MODEL**

An approach to develop simplified dynamic mathematical model for complete mix anaerobic digester for sewage sludge based on Andrews (1969), Hill et al. (1971) & Bello-Mendoza et al. (1998). The developed dynamic model helps to predict the anaerobic digester performance, stability and also compare the growth rate of acidogens and methogens obeying Monod kinetics and also Andrews kinetic model. With the help of the inhibition function developed by Andrews we can also simulate the digester performance by varying pH. Data for predicting the anaerobic digester performance has been taken from Alvarez-Ramirez et al. (2002). The death and endogenous respiration is described by first order kinetics & growth of acidogens and methanogens are based on Monod and Andrews Kinetic Model.

### Dynamic Model for anaerobic digester consists of differential equations developed by material balance around the anaerobic digester digesting sewage sludge

**Mass Balance Equations based on Monod Kinetic Model:**

In the following system of equations the substrate utilization and growth kinetics of methanogens and acidogens is assumed to be dependent upon Monod kinetics Model.
1. Mass balance on the activated sludge biomass requiring hydrolysis
\[
\frac{dX_{V1}^{AS}}{dt} = \frac{X_{V0}^{AS} - X_{V1}^{AS}}{\theta} - k_{DR} X_{V1}^{AS}; \quad X_{V0}^{AS} = 10.0 \text{ g/dm}^3; \quad X_{V1}^{AS}(0) = 11.31 \text{ g/dm}^3
\]
2. Mass balance on particulate solids requiring hydrolysis
\[
\frac{dP_{S1}^H}{dt} = \frac{P_{S0}^H - P_{S1}^H}{\theta} + (1 - Y) k_{DR} X_{V1}^{AS} - k_{H} P_{S1}^H; \quad P_{S0}^H = 1.0 \text{ g/dm}^3; \quad P_{S1}^H(0) = 0.0 \text{ g/dm}^3
\]
3. Mass balance on soluble substrate
\[
\frac{dS_{S1}^A}{dt} = \frac{S_{S0}^A - S_{S1}^A}{\theta} + k_{H} P_{S1}^H + (Y) k_{DR} X_{V1}^{AS} - \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A}; \quad S_{S0}^A = 5.0 \text{ g/dm}^3; \quad S_{S1}^A(0) = 0.0 \text{ g/dm}^3
\]
4. Mass balance on Acidogens
\[
\frac{dX_{1}^A}{dt} = \frac{X_{0}^A - X_{1}^A}{\theta} + Y_{C}^A \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - D_{1} X_{1}^A; \quad X_{0}^A = 0.05 \text{ g/dm}^3; \quad X_{1}^A(0) = 0.1 \text{ g/dm}^3
\]
5. Mass balance on Volatile fatty acid
\[
\frac{dVA_{A1}}{dt} = \frac{VA_{0} - VA_{A1}}{\theta} + \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - Y_{C}^A \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - f_{d} D_{1} X_{1}^A - \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad V_{A0} = 2.0 \text{ g/dm}^3; \quad V_{A1}(0) = 0.0 \text{ g/dm}^3
\]
6. Mass balance on methanogens
\[
\frac{dM_{1}}{dt} = \frac{M_{A1} - M_{1}}{\theta} + \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}} - Y_{C}^M \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad M_{A0} = 0.0 \text{ g/dm}^3; \quad M_{1}(0) = 0.0 \text{ g/dm}^3
\]
7. Mass balance on methane
\[
\frac{dM_{1}}{dt} = \frac{M_{A1} - M_{1}}{\theta} + \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}} - Y_{C}^M \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad M_{A0} = 0.0 \text{ g/dm}^3; \quad M_{1}(0) = 0.0 \text{ g/dm}^3
\]

Mass Balance Equations based on Andrews inhibition Kinetic Model:

In the following system of equations the substrate utilization and growth kinetics of methanogens and acidogens is assumed to be dependent upon Andrews kinetics Model.

1. Mass balance on the activated sludge biomass requiring hydrolysis
\[
\frac{dX_{V1}^{AS}}{dt} = \frac{X_{V0}^{AS} - X_{V1}^{AS}}{\theta} - k_{DR} X_{V1}^{AS}
\]
2. Mass balance on particulate solids requiring hydrolysis
\[
\frac{dP_{S1}^H}{dt} = \frac{P_{S0}^H - P_{S1}^H}{\theta} + (1 - Y) k_{DR} X_{V1}^{AS} - k_{H} P_{S1}^H
\]
3. Mass balance on soluble substrate
\[
\frac{dS_{S1}^A}{dt} = \frac{S_{S0}^A - S_{S1}^A}{\theta} + k_{H} P_{S1}^H + (Y) k_{DR} X_{V1}^{AS} - \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A}; \quad S_{S0}^A = 5.0 \text{ g/dm}^3; \quad S_{S1}^A(0) = 0.0 \text{ g/dm}^3
\]
4. Mass balance on Acidogens
\[
\frac{dX_{1}^A}{dt} = \frac{X_{0}^A - X_{1}^A}{\theta} + Y_{C}^A \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - D_{1} X_{1}^A; \quad X_{0}^A = 0.05 \text{ g/dm}^3; \quad X_{1}^A(0) = 0.1 \text{ g/dm}^3
\]
5. Mass balance on Volatile fatty acid
\[
\frac{dVA_{A1}}{dt} = \frac{VA_{0} - VA_{A1}}{\theta} + \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - Y_{C}^A \frac{k_{S} S_{S1} X_{1}^A}{K_{S}^A + S_{S1}^A} - f_{d} D_{1} X_{1}^A - \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad V_{A0} = 2.0 \text{ g/dm}^3; \quad V_{A1}(0) = 0.0 \text{ g/dm}^3
\]
6. Mass balance on methanogens
\[
\frac{dM_{1}}{dt} = \frac{M_{A1} - M_{1}}{\theta} + \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}} - Y_{C}^M \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad M_{A0} = 0.0 \text{ g/dm}^3; \quad M_{1}(0) = 0.0 \text{ g/dm}^3
\]
7. Mass balance on methane
\[
\frac{dM_{1}}{dt} = \frac{M_{A1} - M_{1}}{\theta} + \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}} - Y_{C}^M \frac{k_{MV} V_{A1} X_{1}^M}{K_{M}^{V} + V_{A1}}; \quad M_{A0} = 0.0 \text{ g/dm}^3; \quad M_{1}(0) = 0.0 \text{ g/dm}^3
\]

VA = HVA + VA, \quad HVA = H^+ + VA^-, \quad HVA = [H^+] [ VA^- ] / Ka

where
\[
X_{V0}^{AS} \quad \text{Concentration of activated sewage sludge biomass in the influent (g/dm}^3\)
$X_{V1}^{AS}$ Concentration of activated sewage sludge biomass in the effluent (g/dm$^3$)
$p_{S0}^H$ Concentration of particulate substrate requiring hydrolysis in influent (g/dm$^3$)
$p_{S1}^H$ Concentration of particulate substrate in effluent (g/dm$^3$)
$s_{S0}^A$ Concentration of soluble substrate in influent (g/dm$^3$)
$s_{S1}^A$ Concentration of soluble substrate in effluent (g/dm$^3$)
$x_{D0}^A$ Concentration of acidogens in influent (g/dm$^3$)
$x_{D1}^A$ Concentration of acidogens in effluent (g/dm$^3$)
$VA_0$ Concentration of volatile fatty acid in influent (g/dm$^3$)
$VA_1$ Concentration of volatile fatty acid in effluent (g/dm$^3$)
$x_{D0}^M$ Concentration of methanogens in influent (g/dm$^3$)
$x_{D1}^M$ Concentration of methanogens in effluent (g/dm$^3$)
M Concentration of methane (g/dm$^3$)
$\Theta$ Hydraulic retention time (days)
HVA Unionized volatile fatty acid concentration (g/dm$^3$)
$VA^-$ Ionized volatile fatty acid concentration (g/dm$^3$)

### Table 1: Kinetic parameter values from literature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{DR}$</td>
<td>Activated sewage sludge biomass death rate coefficient</td>
<td>2.0</td>
<td>Day$^{-1}$</td>
</tr>
<tr>
<td>Y</td>
<td>Biomass degradable fraction</td>
<td>0.3</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>$k^H$</td>
<td>Hydrolysis rate coefficient for acidogens</td>
<td>0.15</td>
<td>Day$^{-1}$</td>
</tr>
<tr>
<td>$k^A$</td>
<td>Maximum specific soluble substrate utilization rate for acidogens</td>
<td>8.0</td>
<td>g/g/day</td>
</tr>
<tr>
<td>$K^A_S$</td>
<td>Half velocity coefficient for acidogenesis</td>
<td>0.045</td>
<td>g/dm$^3$</td>
</tr>
<tr>
<td>$Y^A_C$</td>
<td>Yield coefficient for acidogens</td>
<td>0.2</td>
<td>g/g</td>
</tr>
<tr>
<td>$D^A$</td>
<td>Decay coefficient for acidogens</td>
<td>0.1</td>
<td>Day$^{-1}$</td>
</tr>
<tr>
<td>$f_d$</td>
<td>Net biodegradable fraction</td>
<td>0.73</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>$k^M_V$</td>
<td>Maximum specific substrate utilization rate for methanogens</td>
<td>6.2</td>
<td>g/g/day</td>
</tr>
<tr>
<td>$K^M_C$</td>
<td>Half velocity coefficient for methanogens</td>
<td>0.045</td>
<td>g/dm$^3$</td>
</tr>
<tr>
<td>$D^M$</td>
<td>Decay coefficient for methanogens</td>
<td>0.015</td>
<td>Day$^{-1}$</td>
</tr>
<tr>
<td>$Y^M_C$</td>
<td>Yield coefficient for methanogens</td>
<td>0.057</td>
<td>g/g</td>
</tr>
<tr>
<td>$K_{i1}$</td>
<td>Inhibition coefficient for acidogens</td>
<td>1</td>
<td>g/dm$^3$</td>
</tr>
<tr>
<td>$K_{i2}$</td>
<td>Ionization coefficient for methanogens</td>
<td>0.3</td>
<td>g/dm$^3$</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Ionization constant</td>
<td>$10^{-4.5}$</td>
<td></td>
</tr>
</tbody>
</table>

### COMPUTER SIMULATION

Systems of equations representing the dynamic behaviour of anaerobic digester were solved with the help Polymath. With the help of computer simulation we can predict the effect on the growth rate of acidogens and methanogens; and also on the methane production rate when the digester performance is simulated by using the dynamic model equations with Monod kinetics and Andrews inhibition function.
Figure 2: Anaerobic Digester digesting Sewage Sludge (Monod Kinetics)

Figure 3: Anaerobic Digester digesting Sewage Sludge (Andrews Kinetic Model pH 6)
Figure 4: Anaerobic Digester digesting Sewage Sludge (Andrews Kinetic Model pH 5)

Figure 5: Anaerobic Digester digesting Sewage Sludge (Andrews Kinetic Model pH 4)
RESULTS & DISCUSSION

Computer simulation under transient conditions helps in predicting the digester stability. Figure 2 & 3 represent digester working properly and there is not any build up of volatile fatty acid concentration in the digester and perform properly. The maximum Chemical oxygen demand (COD) removal efficiency is 88% when digester performance is evaluated by Monod Kinetic Model and also maximum methane production rate. Figure 3 represent anaerobic digester behavior at pH 6 using Andrews inhibition function. In this case slightly reduction in COD removal efficiency (84.8%). From Figure 4 when pH drop from 6 to 5 there is build up of volatile fatty acid concentration has been observed and its effects the growth rate of methanogens and also reduction in methane production rate and organic matter removal rate. The COD removal efficiency in this case is 32%. At pH 4 as observed in figure 5 concentration of the volatile acid is very high and reduction of growth rate of methanogens has been observed during early stages and poor efficiency. From the simulated figures we can observe the digester perform properly until the built up of volatile fatty acids inside the digester. Degree of inhibition is a strong function of pH. Build up of volatile fatty acid drop in pH and methanogens can’t survive and hence failure of digester takes place.

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